

EDEXCEL INTERNATIONAL GCSE (9-1)

Further Pure Mathematics 4PM1

GETTING READY TO TEACH

Event code:

First teaching in 2017, first assessment in 2019.



Aims and Objectives

During the training you will:

- Consider the structure, content and assessment of this qualification through examining each of the papers in detail, and the support to guide you through these changes.
- Consider the key changes from 4PM0
- Learn about the new 9 – 1 scale.
- Explore possible teaching and delivery strategies for the new qualification.



Session Agenda

- Welcome
- Overview of the changes to International GCSE Further Pure Mathematics
- Content changes
- Short comfort break
- The new grading scale and resources
- Finish



The logo features a large white circle centered on a teal background. The background is decorated with a repeating pattern of dark teal slanted bars and small dots. Inside the white circle, the text "Pearson Edexcel" is written in a dark teal, sans-serif font.

Pearson Edexcel

About Pearson Edexcel

Pearson is the world's leading learning company. Our mission is to help people make progress in their lives through learning – because we believe that learning opens up opportunities, creating fulfilling careers and better lives.

- ❖ **Qualifications:** our qualifications and assessments help to educate millions of people worldwide.
- ❖ **Support:** we provide innovative textbooks, curriculum materials, multimedia learning tools, IT platforms, professional development.
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- ❖ **Worldwide recognition:** over 150 years of international education experience, more than 3.4 million learners in 70+ countries. Over 9 million scripts marked annually, with exceptionally reliable results.



International GCSE Features



9-1 grading scale

Awarding

- The grading system is changing, but our commitment to awarding grades that accurately reflect learner exam performance remains the same.
- We set new grade boundaries (minimum number of marks needed to achieve each grade) for each assessment of each qualification.

Benefits

- Greater differentiation across levels of attainment, e.g. 2 grades where the current C grade is.
- Rewards truly outstanding achievement with the grade 9.
- Provides more information about student attainment to help progression to A Level.
- Same scale for Pearson Edexcel GCSE and International GCSE allows for clear comparison with English standards, unlike old A* to G grading.



9-1 grading scale

	NEW GRADING STRUCTURE	CURRENT GRADING STRUCTURE
<p>The new grade 9 represents a new level of attainment and has been introduced to differentiate your top performing students.</p> <p>The bottom of the grade 7 broadly aligns with the bottom of the grade A.</p>	9	A*
	8	
	7	A
<p>There's greater differentiation in the middle of the scale, with three new grades 6, 5 and 4 rather than two grades (B and C).</p> <p>The bottom of the grade 4 broadly aligns with the bottom of the grade C.</p>	6	B
	5	C
	4	
	3	D
<p>The bottom of the grade 1 broadly aligns with the bottom of the grade G.</p>	2	E
	1	F
		G
	U	U



World-class qualifications

All Edexcel qualifications are developed to meet Pearson's **World Class Qualification design principles**

Endorsement of educational **thought-leaders and assessment experts** from across the globe

Developed using an understanding and benchmarking of **all educational systems**

Qualifications that support young people to **develop the capabilities** they need to **progress** and prosper in their lives



Supporting transferable skills

- Our transferable skills framework underpins the design all Pearson Edexcel international qualifications and their supporting resources across IPLS, International GCSE and International A Level.
- Ensures our assessments target the skills students' need for successful progression.
- Increasing our support where these skills **naturally** occur through the teaching, learning and assessment.
- Pearson materials and mapping will support you in identifying and developing the acquisition of these skills in students across the full curriculum.
- <https://qualifications.pearson.com/content/dam/pdf/International%20GCSE/General/Transferable-Skills-Information-Pack.pdf>



Assessment Structure



Introduction to the Assessment

Content

There are minor updates to the present specification rather than large scale changes.

Assessment Objectives / Skills Tested

AO1 – Demonstrate a confident knowledge of the techniques of Pure Mathematics.

AO2 – Apply a knowledge of mathematics to the solutions of problems for which an immediate method is not available.

AO3 – Write clear and accurate mathematical solutions.

Structure of Assessment

Both papers are each 50% of the total International GCSE.

Each paper is assessed through a 2 hour examination set and marked by Pearson and consists of around 11 questions with varying mark allocations per question which will be stated on the paper.

The specification will have approximately 40% of the marks distributed evenly over grades 4 and 5, and approximately 60% of the marks distributed evenly over grades 6, 7, 8 and 9.



Relationship of assessment objectives to units

Unit Number	Assessment Objectives		
	AO1	A02	A03
Paper 1	15 – 20%	10 – 15%	17.5 – 25%
Paper 2	15 – 20%	10 – 15%	17.5 – 25%
Total for International GCSE	30 – 40%	20 – 30%	35 – 50%



**Information gained from our
consultations and the changes we are
making**



Information gained from our consultations and the changes we are making.

- International GCSE Maths is valued by teachers and learners as an attractive equivalent to GCSE Maths and an alternative preparation for an IAL or GCE qualification
- Centres would welcome some updates, rather than large-scale changes to the specification
- Centres generally would like to use the 9-1 grade scale



Following consultations we are making the following changes to the International GCSE in Further Pure Mathematics

- A move from the current A*-G to the new 9 – 4 grading structure to maintain comparability with GCSE 9 – 1 Maths
- Some minor additions to the content assessed to reflect this new 9-4 grading structure



but...

- The changes are natural extensions of the current content
- The changes should not involve a large amount of extra teaching time
- Questions requiring the use of problem solving and mathematical reasoning are nothing new to the International GCSE specification – there is just a slight increase in these
- Question types and language used will be very similar to those on the current specification



Marking



Types of marks

M – Method mark; is awarded for a correct method.

Note: the method must be complete of the award of this mark.

A – Accuracy mark; is awarded for a correct answer.

Note: If the method mark has not been awarded, for example and incorrect method has been used, the **A** mark is automatically not available, even if the final answer is correct.

B - Independent mark; is awarded for a correct answer seen.



Advice to candidates (1)

- Candidates should **state** formulae before using them.
- In 'show that' questions **all necessary steps** must be shown.
- The rubric states clearly that answers without any or sufficient working may not gain full marks.
- Candidates should be encouraged to work neatly. This will not only help the candidate organise their thoughts, but help the examiner to see where marks are awarded.



Advice to candidates (2)

- When answering questions that have more than one part, note that part (a) leads into part (b) which leads into part (c) etc.
- Candidates should submit **ONE** attempt at a question. If there are multiple attempts, candidates should indicate clearly which attempt they wish to be marked by crossing out all other attempts.

Note Carefully: Multiple attempts at a question score no marks.



Content changes



1. Logarithmic functions and indices

A	The functions a^x and $\log_b x$ (where b is a natural number greater than one).
B	Use and properties on indices and logarithms, including change of base.
C	Simple manipulation of surds.
D	Rationalising the denominator. (This has been added to the specification)



1. Logarithmic functions and indices

D Rationalising the denominator

We now expect candidates to be able to rationalise the

denominator for expressions such as $\frac{1}{2-\sqrt{3}}$ as well as those

with a denominator that is a pure surd. This brings the specification into line with 4MA1 and 4MB1.

SAM Question 8 part (d) Paper 2

(d) Express $\frac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10} + b$, where a and b are integers.

(2)



Marking Activity

Mark the following 3 candidate responses

Show that $\frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

Show your working clearly. (4)

Mark Scheme

- M1** Method to rationalise $\frac{(\sqrt{12}-1)(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$
- M1** correct expansion of brackets $\frac{2\sqrt{12}-2+\sqrt{12}\sqrt{3}-\sqrt{3}}{4-3}$
- B1** $\sqrt{12} = 2\sqrt{3}$ (may be seen before expansion)
- A1** answer from fully correct working with all steps seen



Responses

that $\frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

A

your working clearly.

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = 4+3\sqrt{3}$$

$$\sqrt{12}-1 = 4+3\sqrt{3} \times (2-\sqrt{3})$$

$$\sqrt{12}-1 = 8-4\sqrt{3}+6\sqrt{3}-9$$

$$\sqrt{12} = -1+2\sqrt{3}$$

$$2\sqrt{3} = -1+1+2\sqrt{3}$$

$$2\sqrt{3} = 2\sqrt{3}$$

Left side = right side

$$\therefore \frac{\sqrt{12}-1}{2-\sqrt{3}} \text{ can be written as } 4+3\sqrt{3}$$

$$\begin{aligned} & \frac{\sqrt{12}-1}{2-\sqrt{3}} \\ &= \frac{2\sqrt{3}-1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \\ &= \frac{6-1}{2-1} = \frac{5}{1} = 5 \end{aligned}$$

$$\frac{\sqrt{12}-1}{2-\sqrt{3}}$$

B

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{2\sqrt{12} + \sqrt{36} - 2 - \sqrt{3}}{2^2 - 3}$$

$$\frac{2\sqrt{12} + \sqrt{36} - 2 - \sqrt{3}}{1}$$

$$= 2\sqrt{4 \times 3} + \sqrt{12}$$

$$= 2\sqrt{4 \times 3} + 6 - 2 - \sqrt{3}$$

$$= 4\sqrt{3} - \sqrt{3} + 4$$

$$= 4\sqrt{3} + 4$$

$$= 4+3\sqrt{3}$$

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = \frac{\sqrt{4 \times 3}-1}{2-\sqrt{3}} = \frac{2\sqrt{3}-1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{3\sqrt{3}}{4-\sqrt{3}}$$

C

$$(\cancel{2\sqrt{3}-1})(2+\sqrt{3}) = 4\sqrt{3} - 2 + 6 - \sqrt{3}$$

$$= 4\sqrt{3} - 2 + 6 - \sqrt{3}$$

$$(2-\sqrt{3})(2+\sqrt{3})$$

$$= 4 - \cancel{2\sqrt{3}} + \cancel{2\sqrt{3}} - 3$$

$$= 4 - 3$$

(Total for Question 10 is 4 marks)



Marking Activity – mark allocation

that $\frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

A

your working clearly.

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = 4+3\sqrt{3}$$

MOM0

$$\sqrt{12}-1 = 4+3\sqrt{3} \times (2-\sqrt{3})$$

$$\sqrt{12}-1 = 8-4\sqrt{3}+6\sqrt{3}-9$$

$$\sqrt{12}-1 = -1+2\sqrt{3}$$

B1

$$2\sqrt{3} = -1+1+2\sqrt{3}$$

$$2\sqrt{3} = 2\sqrt{3}$$

Left side = right side

$\therefore \frac{\sqrt{12}-1}{2-\sqrt{3}}$ can be written as $4+3\sqrt{3}$

A0

B

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = \frac{\sqrt{12}-1 \times (2+\sqrt{3})}{2-\sqrt{3} \times (2+\sqrt{3})}$$

M1

$$= \frac{2\sqrt{12} + \sqrt{36} - 2 - \sqrt{3}}{2^2 - 3}$$

M1

$$= \frac{2\sqrt{12} + \sqrt{36} - 2 + \sqrt{3}}{1}$$

B1

$$= 2\sqrt{4 \times 3} + \sqrt{12}$$

$$= 2\sqrt{4 \times 3} + 6 - 2 - \sqrt{3}$$

A1

$$= 4\sqrt{3} - \sqrt{3} + 4$$

$$= 4\sqrt{3} + 4$$

$$= 4+3\sqrt{3}$$

C

$$\frac{\sqrt{12}-1}{2-\sqrt{3}} = \frac{\sqrt{4 \times 3}-1}{2-\sqrt{3}} = \frac{2\sqrt{3}-1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{3\sqrt{3}}{4-\sqrt{3}}$$

B1

M1

$$(2-\sqrt{3})(2+\sqrt{3}) = 4-3 = 1$$

M0

$$(2\sqrt{3}-1)(2+\sqrt{3}) = 4\sqrt{3}-2+6-\sqrt{3}$$

$$= 4\sqrt{3}-\sqrt{3}-2+6$$

$$= 4\sqrt{3}-\sqrt{3}+4$$

A0

(Total for Question 10 is 4 marks)



1. Logarithmic functions and indices

SAM Question 6 Paper 2

6 Solve the equation $\log_2 x + 6\log_x 2 = 7$

Candidates are expected to be able to use the properties of indices and logarithms, including change of base of the logarithm.

This question will develop into a quadratic equation so that specification reference **2A** (manipulation of quadratic equations) will also be tested here.



1. Logarithmic functions and indices

At its very simplest, questions such as could be asked.

Find the value of

$$\log_3 9$$

This part is a 1 mark question as it is expected that candidates are able to 'write down' the correct value.

(Paper 2 June 2015 Q10 (a))



1. Logarithmic functions and indices

This is an example of a 'show' question

(c) 'Show that'

$$2x \log_3 x - 3 \log_3 x - 4x \log_9 4 + 6 \log_9 4 = \log_3 \left(\frac{x}{4} \right)^{(2x-3)} \quad (6)$$

The instruction 'show that' means that candidates must show EVERY step in their working, and note that there are 6 marks available in this part. This is an indication that a lot of work is required for that many marks. Candidates must demonstrate they understand logarithmic rules.

(Paper 2 June 2015 Q10 (c))



1. Logarithmic functions and indices

The final part of the same question now develops using the word 'hence'. This implies that the previous result is to be used to solve the equation.

(d) Hence solve the equation

$$2x \log_3 x - 3 \log_3 x - 4x \log_9 4 + 6 \log_9 4 = 0$$

This means, solve the equation $\log_3 \left(\frac{x}{4} \right)^{(2x-3)} = 0$

Solution and working are on the next slide.



1. Logarithmic functions and indices

Solving the equation $\log_3 \left(\frac{x}{4} \right)^{(2x-3)} = 0$

Use the laws of logs first

Consider **both** possibilities

$$(2x-3) \log_3 \left(\frac{x}{4} \right) = 0 \Rightarrow 2x-3=0 \text{ or } \log_3 \left(\frac{x}{4} \right) = 0$$

$$\Rightarrow x = \frac{3}{2} \text{ or } \frac{x}{4} = 3^0 \Rightarrow x = 4$$

$$\Rightarrow x = \frac{3}{2}, 4$$

There are two roots !



2. The Quadratic Function

There is no new content in this section.

A	The manipulation of quadratic expressions.
B	The roots of a quadratic equation.
C	Simple examples involving functions of the roots of a quadratic equation.



2. The Quadratic Function

A Manipulation of quadratic expressions

Candidates are expected to be able to recognise applications where quadratic equations require to be solved as part of the problem.

For example; **SAMs Question 4 (a) Paper 1**

A particle P is moving along the x -axis.

At time t seconds ($t \geq 0$) the velocity, v m/s, of P is given by $v = 4t^2 - 19t + 12$

(a) Find the values of t for which P is instantaneously at rest.

(2)



2. The Quadratic Function

B The roots of a quadratic equation

Candidates should be able to use the discriminant to identify whether the roots are equal, real or not real as in part (c) of this question in the SAMs.

SAMs Question 4 part (c) Paper 2

$$f(x) = 2x^3 + px^2 + qx + 12 \quad p, q \in \mathbb{Z}$$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of q

(6)

(b) hence factorise $f(x)$ completely

(2)

(c) show that the equation $f(x) = 0$ has only one real root.



2. The Quadratic Function

Completing the square of a quadratic function

The questions can be simple, or as in the following example, the level of demand has been increased by including a negative coefficient of x^2 in the function.

$$f(x) = 4 + 3x - x^2$$

(a) Write $f(x)$ in the form $P - Q(x + R)^2$



2. The Quadratic Function

Completing the square of a quadratic function

Candidates are also expected to interpret their answer and the questions goes on to ask:

The curve C has equation

$$y = 4 + 3x - x^2$$

(b) Find the coordinates of the maximum point of C . (1)

Candidates should look at the marks being awarded to be aware of the amount of work involved. In this case, there is only 1 mark available, so this is a strong hint that the answer can be written down using the result from part (a).



2. The Quadratic Function

An example of a candidate's response

In this response, the candidate has not interpreted their answer from (a), but has differentiated (correctly) to find the coordinates.

This is still only worth one mark.

The curve C has equation $y = 4 + 3x - x^2$

(b) Find the coordinates of the maximum point of C .

$$(b) \frac{dy}{dx} = 3 - 2x$$

$$\text{put } \frac{dy}{dx} = 0$$

$$\therefore \text{so, } 3 - 2x = 0$$

$$\therefore x = \frac{3}{2}$$

$$\text{put } x = \frac{3}{2} \text{ in } y = 4 + 3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2$$
$$= \frac{25}{4}$$

$$\text{Maximum point} = \left(\frac{3}{2}, \frac{25}{4}\right)$$



2. The Quadratic Function

C Simple examples involving functions of the roots of a quadratic equation.

Students are expected to understand and use;

the equation $ax^2 + bx + c = 0$ has roots α and β

such that $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

The most common source of error in this topic is the inability to expand polynomials correctly and this is pre-requisite in this topic.



2. The Quadratic Function

C Simple examples involving functions of the roots of a quadratic equation.

SAMs Question 9 Paper 1

9 The roots of a quadratic equation are α and β where $\alpha + \beta = -\frac{7}{3}$ and $\alpha\beta = -2$

(a) Find a quadratic equation, with integer coefficients, which has roots α and β

(4)

Given that $\alpha > \beta$ and without solving the equation,

(b) show that $\alpha - \beta = \frac{11}{3}$

(2)

(c) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha + \beta}{\alpha} \text{ and } \frac{\alpha - \beta}{\beta}$$

(7)



2. The Quadratic Function

C Simple examples involving functions of the roots of a quadratic equation.

Two common errors seen

$$\begin{aligned} f(x) &= 196x^2 + 4774x + C \\ &= 196x^2 + 4774x + 7 \\ &= 28x^2 + 682x + 1 \end{aligned}$$

This is not an equation!
An equation must $= 0$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - \alpha\beta$$

An example of
careless algebra



3. Identities and Inequalities

There is no new content here

A	Simple algebraic division
B	The factor and remainder theorems
C	Solutions of equations, extended to include the simultaneous solution of one linear and one quadratic in two variables
D	Simple inequalities, linear and quadratic
E	The graphical representation of linear inequalities in two variables



3. Identities and Inequalities

SAMs Question 4 Paper 1

4

$$f(x) = 2x^3 + px^2 + qx + 12 \quad p, q \in \mathbb{Z}$$

Given that $(x + 3)$ is a factor of $f(x)$ and that when $f'(x)$ is divided by $(x + 3)$ the remainder is 37

(a) show that $p = 1$ and find the value of q

(6)

(b) hence factorise $f(x)$ completely

(2)

(c) show that the equation $f(x) = 0$ has only one real root.

(2)

This question also brings in specification reference **9A**
(differentiation)



3. Identities and Inequalities

Here is a quote from the Principal Examiners report for this question.

Extracts from the Principal Examiner's report:

- Most candidates realised that the equations had to be solved simultaneously and very many candidates obtained full marks in this question. Solutions usually took the simpler option of eliminating y . Solving the resulting quadratic equation was well understood though the formula method was commonly seen.
- A number of candidates chose to eliminate x and obtain an equation in y . This resulted in greater algebraic manipulation and accordingly this approach was generally less successful. Some stopped at $y = y^2 - 16y + 60$ then solved just the right hand side $= 0$.

3. Identities and Inequalities

2	$x^2 - 6x + 5 = 11 - x$	M1	
	$x^2 - 5x - 6 = 0$ OR $y^2 - 17y + 60 = 0$	A1	
	$(x - 6)(x + 1) = 0$ $(y - 12)(y - 5) = 0$	dM1	
	$x = 6, y = 5$	A1	
	$x = -1, y = 12$	A1	[5]

M1 Obtain an equation in one variable. Must be quadratic but no simplification needed

A1 Correct simplified 3 term quadratic equation, terms in any order

dM1 Solve their quadratic by any valid means (see "General Principles")

A1 Either (x, y) pair correct or both x values or both y values correct

A1 Second pair correct. It must be clear how the values are paired. (Horizontally as shown or vertically is sufficient.)

Marking Activity 2

Mark these student responses using the detailed mark scheme

Student response 1

$$\begin{aligned}
 y+x &= 11 \\
 y &= 11-x \\
 11-x &= x^2-6x+5 \\
 x^2-6x+x+5-11 &= 0 \\
 x^2+5x-6 &= 0 & \begin{array}{l} -6 \\ -1 \uparrow 6 \end{array} \\
 (x-1)(x+6) \\
 x=1 & \quad x=-6 \\
 \text{if } x=1 & \quad \text{if } x=-6 \\
 y=11-1 & \quad y=11+6 \\
 y=10 & \quad y=17
 \end{aligned}$$

Student Response 2

$$\begin{aligned}
 y &= x^2-6x+5 \\
 x^2-5x-x+5 \\
 x(x-5)-1(x-5) \\
 y &= (x-1)(x-5) \\
 y+x &= 11 \\
 x &= 11-y \\
 y &= (11-y)^2-6(11-y)+5 \\
 &= (11-y)(11-y)-66+6y+5 \\
 &= 121-11y-11y+y^2-66+6y+5 \\
 &= 60-16y+y^2 \\
 x-1 &= 0 \quad \text{or} \quad x-5=0 \\
 x=1 & \quad \text{or} \quad x=5 \\
 \text{substitute } x & \text{ to find } y \\
 y+x &= 11 \\
 y+1 &= 11 \quad \text{or} \quad y+5=11 \\
 y &= 11-1 \quad \text{or} \quad y=11-5 \\
 y &= 10 \quad \text{or} \quad y=6 \\
 x=1, & \quad y=10 \\
 x=5, & \quad y=6
 \end{aligned}$$

Marking Activity 2

Mark these student responses using the detailed mark scheme

Student response 1

$$\begin{aligned}
 y+x &= 11 \\
 y &= 11-x \\
 11-x &= x^2-6x+5 \\
 x^2-6x+x+5-11 &= 0 & \text{M1} \\
 x^2+5x-6 &= 0 & \text{A0} \\
 (x-1)(x+6) & & \text{dM1} \\
 x=1 & \quad x=-6 \\
 \text{if } x=1 & \quad \text{if } x=-6 \\
 y=11-1 & \quad y=11+6 & \text{A0} \\
 y=10 & \quad y=17 & \text{A0}
 \end{aligned}$$

Student Response 2

$$\begin{aligned}
 y &= x^2-6x+5 & \text{M0} \\
 x^2-5x-x+5 & \\
 x(x-5)-1(x-5) & \\
 y &= (x-1)(x-5) & \text{A0} \\
 y+x &= 11 \\
 x &= 11-y \\
 y &= (11-y)^2-6(11-y)+5 \\
 &= (11-y)(11-y)-66+6y+5 \\
 &= 121-11y-11y+y^2-66+6y+5 \\
 &= 60-16y+y^2 \\
 x-1 &= 0 \quad \text{or} \quad x-5=0 & \text{dM0} \\
 x=1 & \quad \text{or} \quad x=5 \\
 \text{substitute } x & \text{ to find } y \\
 y+x &= 11 \\
 y+1 &= 11 \quad \text{or} \quad y+5=11 \\
 y &= 11-1 \quad \text{or} \quad y=11-5 \\
 y &= 10 \quad \text{or} \quad y=6 \\
 x=1, y=10 & & \text{A0 A0} \\
 x=5, y=6 &
 \end{aligned}$$

3. Identities and Inequalities

SAMs Question 2 Paper 2

In this question, candidates have to find a simple linear inequality in part (a), a quadratic equality in part (b), and combine the result for part (c).

2 Find the set of values of x for which

(a) $3 + x < 2x - 1$ (1)

(b) $x(x - 1) > 6$ (3)

(c) **both** $3 + x < 2x - 1$ **and** $x(x - 1) > 6$ (1)



3. Identities and Inequalities

Teaching quadratic inequalities (1)

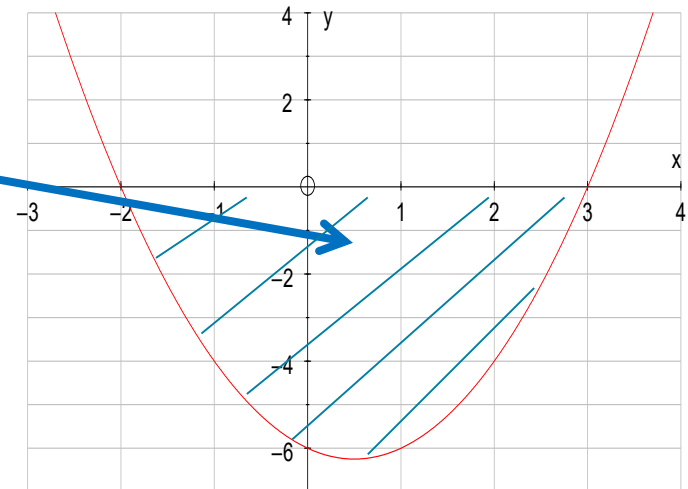
e.g. $x^2 - x - 6 < 0$

.Find critical values: solve $x^2 - x - 6 < 0 \Rightarrow x = 3, x = -2$

.Sketch or table?

.So $-2 < x < 3$

For $x^2 - x - 6 > 0$
 $x < -2$ OR $x > 3$



Do Not Write $3 < x < -2$



3. Identities and Inequalities

Teaching quadratic inequalities – (2)

Using a table

	$x < -2$	$-2 < x < 3$	$x > 3$
$(x - 3)$	—	—	+
$(x + 2)$	—	+	+
$(x - 3)(x + 2)$	+	—	+

So for $x^2 - x - 6 > 0$ $x < -2$ OR $x > 3$

Or for $x^2 - x - 6 < 0$ $-2 < x < 3$

Another neat way for quadratic inequalities

	< 0	> 0
$+x^2$	$-2 < x < 3$	$x > 3$ OR $x < -2$
$-x^2$		

So for $x^2 - x - 6 > 0$ $x < -2$ OR $x > 3$

Or for $x^2 - x - 6 < 0$ $-2 < x < 3$



3. Identities and Inequalities

SAMs Question 1 Paper 1

Candidates are expected to be able to plot straight lines from the equation of a straight line given in any form, and to identify a region defined by inequalities.

- 1 (a) On the axes below, sketch the lines with equations $2x + 3y = 8$ and $2y = 4x + 1$

On your sketch, show the coordinates of the points where the lines cross the coordinate axes. (2)

- (b) Show, by shading on your sketch, the region R defined by the inequalities

$$2x + 3y \leq 8 \quad 2y \leq 4x + 1 \quad y \geq 0 \quad x \leq 2$$

(2)



5. Graphs

There is no new content in this section.

A

Graphs of polynomials and rational functions with linear denominators

B

The solution of equations and transcendental functions by graphical methods



5. Graphs

SAMs Question 7 Paper 2

The concept of asymptotes parallel to the coordinate axes is expected as in this question.

The curve C with equation

$$y = \frac{ax - 5}{x - b}$$

where a and b are integers, crosses the x -axis at the point $(2.5, 0)$. The asymptote to C which is parallel to the y -axis has equation $x = 1$

(a) (i) Show that $a = 2$

(ii) Find the value of b .

(3)

(b) Find the coordinates of the point where C crosses the y -axis.

(1)

(c) Find the equation of the asymptote to C which is parallel to the x -axis.

(1)

(d) Using the axes below, sketch the curve C showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

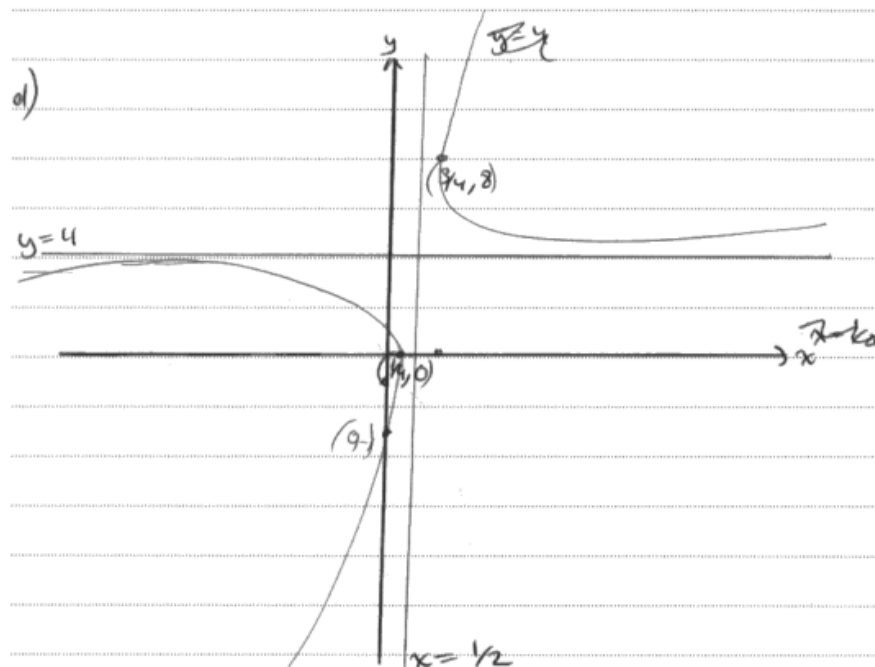


5. Graphs

This is a typical sketch of a student response. (Question 10 (e) Paper 1
Note how poorly the curve endings are drawn.

The asymptote drawn parallel to the y-axis is correct and gained B1

There is no asymptote parallel to the x-axis, so the line drawn here was ignored as the candidate was already penalised for an incorrect position of the curve.



5. Graphs

SAMs Question 7 Paper 1

This question follows on the next slide.

Part (c) of the question links in to specification reference **1B**



5. Graphs

7 (a) Complete the table of values for

$$y = 2^{\left(\frac{x}{2}+1\right)} + 1$$

giving your answers to 2 decimal places where appropriate.

(2)

x	0	1	2	3	4	5
y	3				9	12.31

(b) On the grid opposite, draw the graph of $y = 2^{\left(\frac{x}{2}+1\right)} + 1$ for $0 \leq x \leq 5$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to 1 decimal place, of the root of the equation $\log_2(4x - 6)^2 - x = 2$ in the interval $0 \leq x \leq 5$

(4)



5. Series

There is no new content here, but the formulae for the sum to n terms of an Arithmetic series and of a Geometric series will now be given in the formula sheet. The formula for the sum to infinity of a geometric series will also be given. (The formulae for the n th terms will **NOT** be given).

A	Use of the Σ notation
B	Arithmetic and Geometric series

Use of the sum to infinity of a convergent series, including the use of $|r| < 1$ is required.

Note: Proofs of the summation formulae are not required – although they are beautiful mathematics and are well worth discussing with students.



5. Series

Arithmetic Series

SAMs Question 8 Paper1

8 The sum S_n of the first n terms of an arithmetic series is given by $S_n = 2n(n + 3)$

(a) Find the first term of the series.

(1)

(b) Find the common difference of the series.

(2)

The n th term of the series is T_n

Given that $6S_{(n-4)} = 7T_{(n+3)}$

(c) find the value of n .

(6)



5. Series

Arithmetic Series

SAMs Question 8 Paper1

8 The sum S_n of the first n terms of an arithmetic series is given by $S_n = 2n(n + 3)$

(a) Find the first term of the series.

(b) Find the common difference of the series.

A common error in a question in this form

The first term is found easily, because $S_n = U_n$

However, $S_2 = U_1 + U_2$ so $U_2 = S_2 - U_1$



5. Series

Geometric Series

SAMs Question 1 Paper 2

1 The n th term of a geometric series is $3e^{(1-2n)}$

Find the sum to infinity of this series.

Give your answer in the form $\frac{ae}{e^b - 1}$ where a and b are integers to be found.

This is an example of an AO2 question.

In order to start it is necessary to find the first term and the common ratio as shown on the next slide, although no indication of this is given in the question.



5. Series

It is always useful to write out a few terms of the sum of a series given in Σ form.

For example;

n	1	2	3	4	5
U_n	$3e^{-1}$	$3e^{-3}$	$3e^{-5}$	$3e^{-7}$	$3e^{-9}$

$$\Rightarrow \text{First term} = 3e^{-1}$$

$$\Rightarrow \text{Common ratio} = \frac{3e^{-3}}{3e^{-1}} = e^{-2}$$



5. Series

The question asks for the sum to infinity in a specific form, so candidates should always look at the given form at every stage to make sure they are working in the correct direction. The solution below is of course only one way this could be simplified.

$$S = \frac{3e^{-1}}{1 - e^{-2}} = \frac{\frac{3}{e}}{1 - \frac{1}{e^2}} = \frac{\frac{3}{e}}{\frac{e^2 - 1}{e^2}} = \frac{3e}{e^2 - 1}$$



6. The Binomial Expansion

There are no changes in this section but the formula for the Binomial expansion of $(1+x)^n$ will now be given in the formula sheet.

A Use of the binomial series $(1+x)^n$

Questions can be set for the use of the series when:

- (i) n is a positive integer
- (ii) n is rational and $|x| < 1$

The validity condition for (ii) is expected.



6. The Binomial Expansion

SAMs Question 8 Paper 2

This question is on the next slide.

It includes specification reference **1C** and **1D** in parts (c) and (d)



6. The Binomial Expansion

- 8 (a) Expand $\frac{3}{\sqrt{1-2x}}$ in ascending powers of x up to and including the term in x^3 and simplifying each term as far as possible.

(4)

- (b) Write down the range of values of x for which this expansion is valid.

(1)

- (c) Show that $\frac{3}{\sqrt{0.9}} = \sqrt{10}$

(1)

- (d) Express $\frac{1}{\sqrt{10}-3}$ in the form $a\sqrt{10} + b$, where a and b are integers.

(2)

- (e) Hence, using your expansion with a suitable value for x , obtain an approximation to 5

decimal places of $\frac{1}{\sqrt{10}-3}$

(3)



6. The Binomial Expansion

Question 6 Paper 2 June 2017

$$f(x) = (p + qx)^6 \text{ where } p \neq 0 \text{ and } q \neq 0$$

- (a) Find the expansion of $f(x)$ in ascending powers of x up to and including the term in x^4 , simplifying each term as far as possible. (3)

In the expansion of $f(x)$, 4 times the coefficient of x^4 is equal to 9 times the coefficient of x^2

Given that $(p + q) > 0$ and $f(1) = 15625$

- (b) find the possible pairs of values of p and q . (6)

6. The Binomial Expansion

$$f(x) = (p + qx)^6 \text{ where } p \neq 0 \text{ and } q \neq 0$$

$$= p^6 \left(1 + \frac{6q}{p}x + \frac{15q^2}{p^2}x^2 + \frac{20q^3}{p^3}x^3 + \frac{15q^4}{p^4}x^4 \right)$$

This is correct,
but has been
overcomplicated

Extracts from the Principal Examiner's report on part (a):

- In part (a) many candidates started their expansion with a '1', missed out the binomial coefficients 6,15,20, or gave an expansion in terms of powers of q or with no terms in p . Some did not simplify their expansion, leaving the brackets around $(qx)^2$ and other powers.

7. Scalar and vector quantities

There are no changes in this section

A	The addition and subtraction of coplanar vectors and the multiplication of a vector by a scalar.
B	Components and resolved parts of a vector
C	Magnitude of a vector
D	Position vector
E	Unit vector
F	Use of vectors to establish simple geometrical properties of geometrical figures.



7. Scalar and vector quantities

SAMs Question 3 Paper 2

3 O , A and B are fixed points such that

$$\vec{OA} = 4\mathbf{i} + 3\mathbf{j} \quad \vec{OB} = 8\mathbf{i} + p\mathbf{j} \quad \text{and} \quad |\vec{AB}| = 2\sqrt{13}$$

(a) Find the possible values of p .

(3)

Given that $p > 0$

(b) find a unit vector parallel to \vec{AB}

(2)



7. Scalar and vector quantities

Vector questions always clearly differentiate between the strongest and the weakest candidates.

Many marks in questions in vectors are lost due to two reasons;

1. candidates do not have a consistent approach to directions which are so crucial in this work, and
2. candidates use poor notation.

Questions frequently begin with fairly routine demands.

The next slide gives a typical example.



7. Scalar and vector quantities

Example Question January 2016

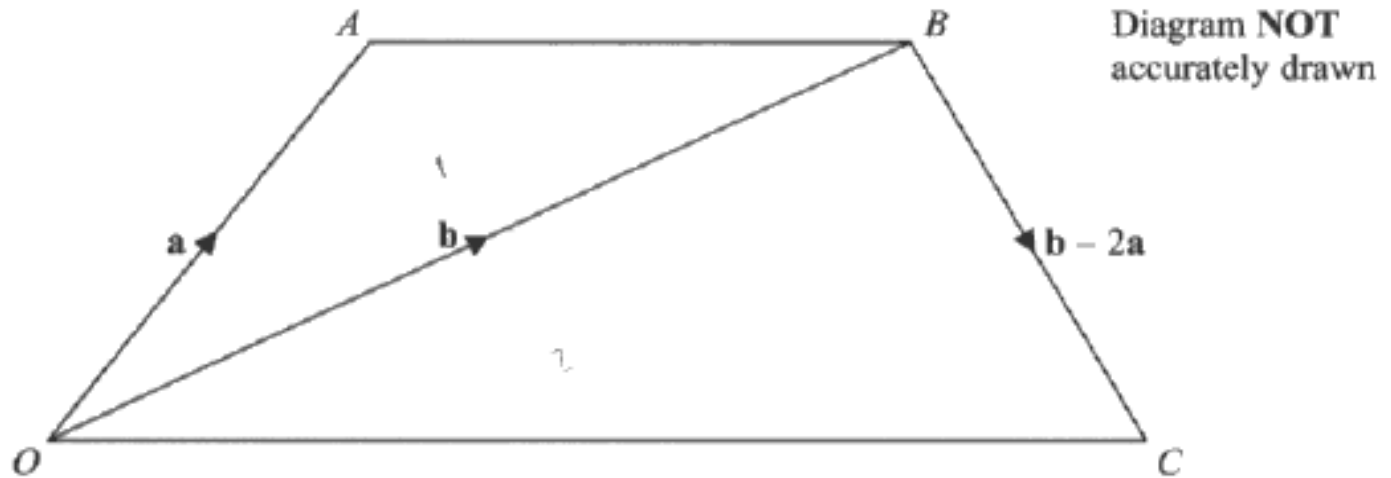


Figure 2

Figure 2 shows a quadrilateral $OACB$

$$\vec{OA} = \mathbf{a}, \vec{OB} = \mathbf{b} \text{ and } \vec{BC} = \mathbf{b} - 2\mathbf{a}$$

(a) (i) Prove that \vec{AB} is parallel to \vec{OC}

(ii) Show that $AB:OC = 1:2$



7. Scalar and vector quantities

January 2016

It is always advisable to establish correct vector statements **first**.
This will help the candidate **AND** crucially score the Method mark.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} \Rightarrow -\mathbf{a} + \mathbf{b}$$

$$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} \Rightarrow \mathbf{b} + \mathbf{b} - 2\mathbf{a} = 2(-\mathbf{a} + \mathbf{b})$$

In a proof, a conclusion is required;

$$\overrightarrow{OC} = 2\overrightarrow{AB} \text{ so same direction,}$$

$$\text{and the ratio is } \overrightarrow{AB} : \overrightarrow{OC} = 1 : 2$$



7. Scalar and vector quantities

January 2016

The second part of the question increases the level of demand and it continues with;

The point D lies on OB such that $OD:DB = 2:3$

(b) Find the ratio of the area of $\triangle ODC$ to the area of $\triangle OAB$.

(6)

These parts of vector questions are usually amongst the most challenging in the whole paper, but they need not be so.

The most common error in vector questions involving length or area is to attempt to use vectors instead of lengths.

In this case, the solution is straightforward when areas of triangles are used;

either $\frac{1}{2}ab \sin C$ or half \times base \times height.



8. Rectangular Cartesian coordinates

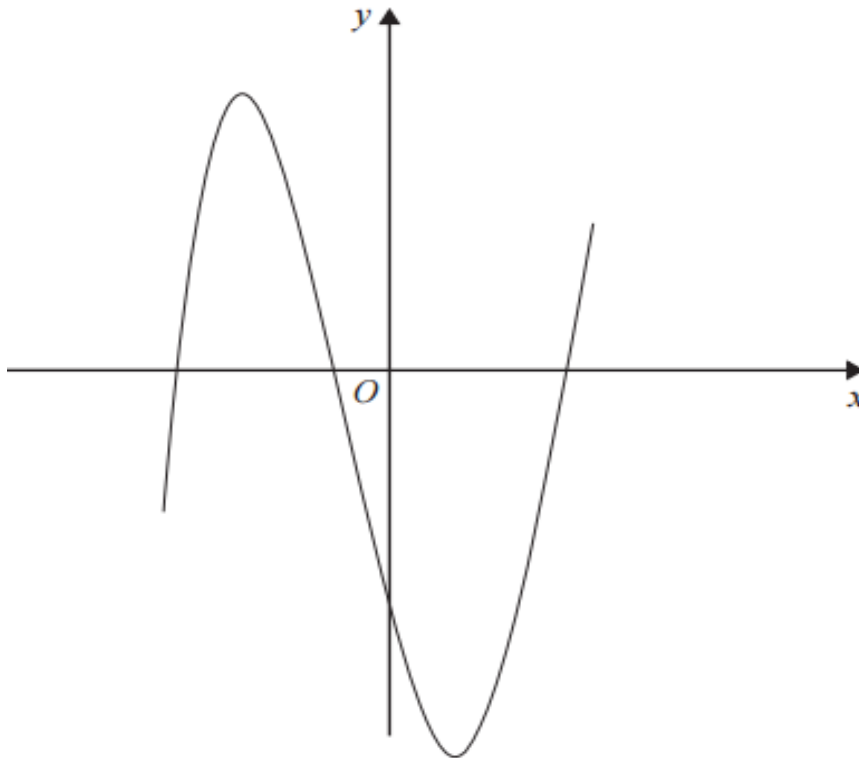
There are no changes in this section

A	The distance between two points.
B	The point dividing a line in a given ratio.
C	Gradient of a straight line joining two points.
D	The straight line and its equation.
E	The condition for two lines to be parallel or to be perpendicular



8. Rectangular Cartesian coordinates

SAMs Question 10 Paper 2



The question follows on the next slide

Figure 1



8. Rectangular Cartesian coordinates

SAMs Question 10 Paper 2

Figure 1 shows the curve M with equation $y = x^3 - 13x - 12$

The point P , with x coordinate -2 , lies on M and line l_1 is the tangent to M at the point P .

(a) Find an equation for l_1 (5)

The point Q lies on M and the line l_2 is the tangent to M at the point Q .

Given that l_1 and l_2 are parallel,

(b) find an equation for l_2 (4)

The normal to M at P meets l_2 at the point R .

(c) Find the coordinates of R . (4)

(d) Find the exact length of the line PR . (2)

The tangent and normal at P and the tangent and normal at Q form a rectangle.

(e) Find the exact area of this rectangle. (3)



8. Rectangular Cartesian coordinates

These questions are always generally well attempted.

Key points which candidates would do well to note.

1. **Always draw a sketch**, or annotate and add the sketch if one is given in the question. A common feature of poor attempts at these questions is the lack of a good careful sketch.
2. Dividing a line in a given ratio – those candidates who attempt to use the formula often make mistakes. Those who use similar triangles nearly always get it right.



8. Rectangular Cartesian coordinates

And finally:

Many candidates do not read questions carefully and find the equation of the normal when the tangent is required, and vice versa.

Please encourage your students to read these questions very carefully to avoid losing marks needlessly.



9. Calculus

There are no changes in this section.

However, the formula for Quotient rule will now be given in the formula sheet.

The formulae for Product and Chain rules will be expected to be known.



9. Calculus

A	Differentiation and integration of sums and multiples of powers of x , $\sin ax$, $\cos ax$ (excluding integration of $1/x$)
B	Differentiation of a product, quotient and simple cases of a function of a function.
C	Applications to simple kinematics and to determination of areas and volumes
D	Stationary points and turning points
E	Maxima and minima
F	The equations of tangents and normals to the curve $y = f(x)$
G	Applications of calculus to rates of change and connected rates of change



9. Calculus

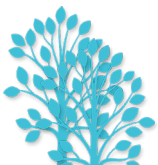
SAMs Question 6 Paper 1

An example of a question involving specification reference **9A** and **9B**

$$y = e^x (x^2 - 3x)$$

Show that $y - 2\frac{dy}{dx} + \frac{d^2y}{dx^2} = 2e^x$

Note this is another example of a **show** question. Encourage your students to show every step of working.



9. Calculus

SAMs Question 4 Paper 2 Spec 9C – Applications to kinematics

A particle P is moving along the x -axis.

At time t seconds ($t \geq 0$) the velocity, v m/s, of P is given by $v = 4t^2 - 19t + 12$

(a) Find the values of t for which P is instantaneously at rest.

When $t = 0$, the displacement of P from the origin is -4 m.

(b) Find the displacement of P from the origin when $t = 6$

At time t seconds the acceleration of P is a m/s².

(c) Find the value of t when $a = 0$

Note: When integrating the velocity to find the displacement, a very common error is to omit $+c$



9. Calculus – Integration

Key point – the constant of integration

- Don't forget to include +c in indefinite integrals. This could result in the loss of just a single mark in a question such as.

$$f(x) = 3x^2 + \frac{5}{x} + 3 \quad \text{find } \int f(x) \, dx$$

The level of demand could be increased by asking candidates to find the equation of a curve given also the coordinates of a point on the curve, in which case several marks would be lost by this omission.



9. Calculus

SAMs

Question 11

Paper 1

Part (a) is an example
of a question aiming
for Grade 9

Diagram **NOT**
accurately drawn

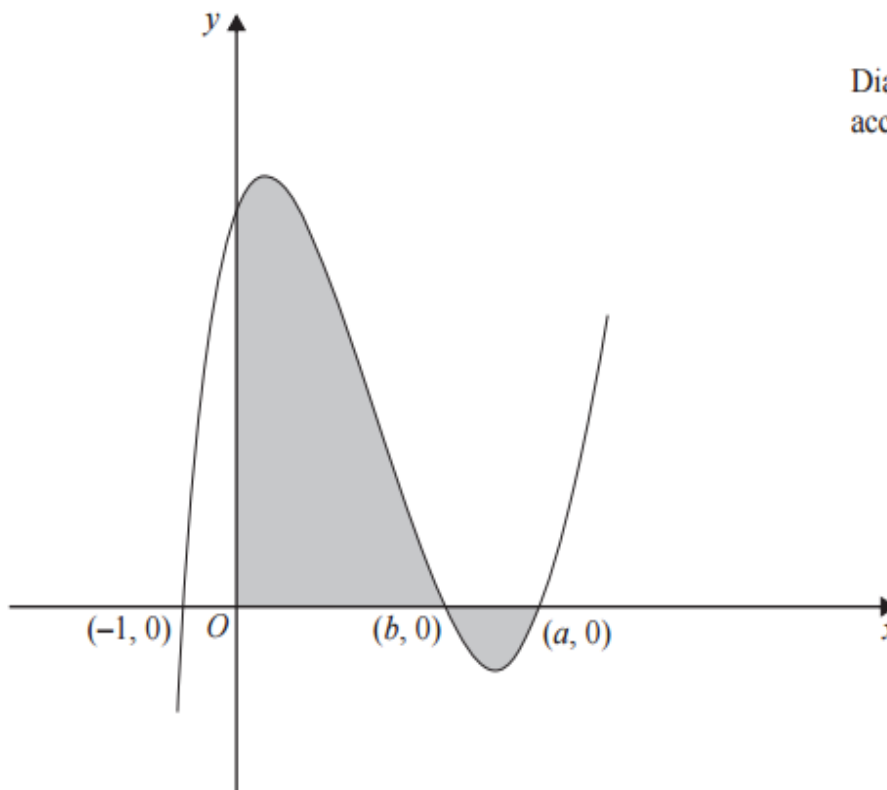


Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$, which passes through the points with coordinates $(-1, 0)$, $(b, 0)$ and $(a, 0)$ where $0 < b < a$.

Given that $f'(x) = 6x^2 - 26x + 12$

(a) find,

- (i) the value of a ,
- (ii) the value of b .

(8)

(b) Use algebraic integration to determine the exact value of the total area of the shaded regions shown in Figure 3.



9. Calculus – Integration

Key point - draw a sketch to find an area or a volume

Some questions include a sketch and some do not as in the following example.

The region enclosed by the curve with equation $y = 4x^2 - 9$, the positive x -axis and the negative y -axis is rotated through 360° about the x -axis.

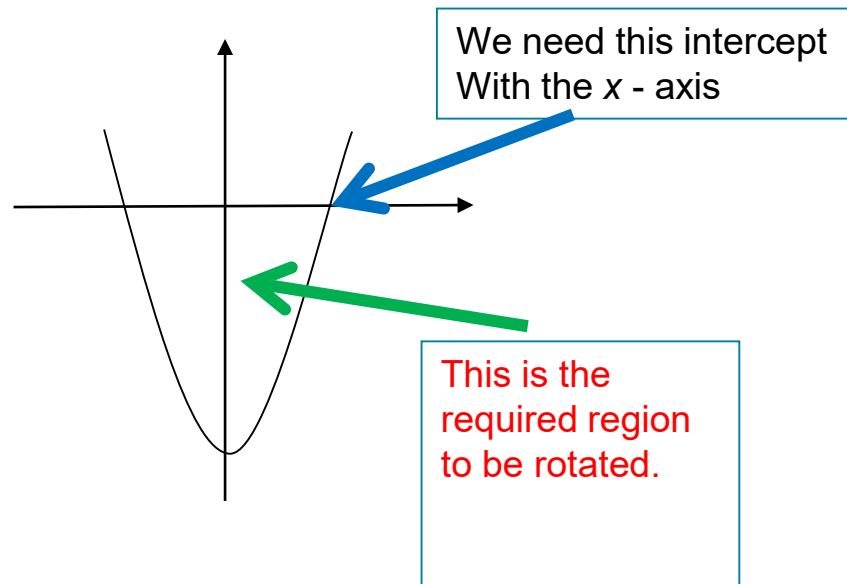
Use algebraic integration to find, to 3 significant figures, the volume of the solid generated.



9. Calculus – Integration

Key point - draw a sketch to find an area or a volume

The question does not give any limits, so a thumbnail sketch would quickly establish what is required to be found.



9. Calculus – Integration

This question is testing volumes of revolution, so a method mark for increasing the power of a variable by 1 was not available. The method marks were dependent on candidates using the correct formula and method for determining the volume.

Step 1 – find the x intercepts. $4x^2 - 9 = 0 \Rightarrow x = \pm \frac{3}{2}$ M1A1

Step 2 – establish limits of integration (from sketch) $\frac{3}{2}$ and 0

Step 3 – set up integration $\text{Vol} = \pi \int_0^{\frac{3}{2}} (4x^2 - 9)^2 dx$ M1

Step 4 – integrate $V = \pi \int_0^{\frac{3}{2}} 16x^4 - 72x^2 + 81 dx = \pi \left[\frac{16}{5}x^5 - 24x^3 + 81x \right]$ M1d

Step 5 – evaluate $V = \frac{324\pi}{5} = 203.57... \approx 204$ (3sf) A1 (5)

Both method marks are available even if π is omitted.



9. Calculus – Integration

This was the PE report on this question.

Question 1

‘This question proved be a difficult beginning for quite a few candidates, as many did not know the correct formula for the volume of revolution, and as the second method mark was dependent on the first, a significant majority of the entry achieved only the first mark for finding $x = \pm \frac{3}{2}$

Even when candidates knew the formula, many put in the lower limit as $-\frac{3}{2}$

leading to the incorrect answer. In this question, **a simple sketch of a correctly placed curve of the quadratic, with the correct area to be rotated shaded (given clearly in the question), would have been of great use in identifying the required region’.**

9. Calculus

SAMs Question 5 Paper 1

Two numbers x and y are such that $2x + y = 13$

The sum of the squares of $2x$ and y is S .

(a) Show that $S = 8x^2 - 52x + 169$

(3)

Using calculus,

(b) find the value of x for which S is a minimum, justifying that this value of x gives a minimum value for S .

(4)

(c) find the minimum value of S .

(2)

Spec 9E – Maxima and minima may be set in the context of a practical problem. In this case this is set in the context of an algebraic problem



9. Calculus

SAMs Question 11 Paper 2

Spec 9G Application of calculus to
connected rates of change

11

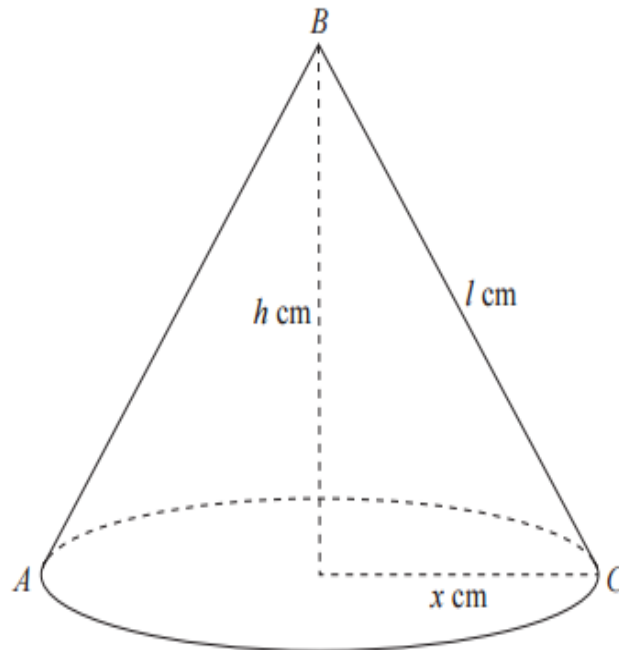


Figure 2

Diagram NOT
accurately drawn

This is an example
of a question aiming
for Grade 9

The question follows
on the next slide



9. Calculus - differentiation

SAMs Question 11 Paper 2

Figure 2 shows a right circular cone with a base radius of x cm. The slant height of the cone is l cm and the height of the cone is h cm. The vertex of the cone is B and the points A and C , on the base of the cone, are such that AC is a diameter of the base.

The cone is increasing in size in such a way that the size of the angle ABC is constant at 60° and the **total** surface area of the cone is increasing at a constant rate of $10 \text{ cm}^2/\text{s}$.

Find the exact rate of increase of the volume of the cone when $x = 6$

(11)

The formula for the curved surface area of a cone is given in the formula sheet. This question also includes specification **10C** Trigonometry
The model answer to this question is given in the next two slides.



9. Calculus – connected rates of change

This question is usually not well answered at all, here is a model answer in the next slides together with the allocation of marks. In this question we have the surface area given in terms of the slant height in the formula sheet.

Note: Common error! All differentiation must be in terms of one variable. Differentiating the volume of a cone with respect to r will gain M0 if there is an h present in dV/dr !

$$V = \frac{1}{3}\pi r^2 h$$
$$\frac{dV}{dr} = \frac{2\pi r}{3}$$

This gains no marks!



9. Calculus - differentiation

Mark scheme for SAMs Question 11 Paper 2

Question	Working	Answer	Mark
11	<p>Total Surface Area of the cone</p> $l = \frac{x}{\sin 30^\circ} = 2x$ $A = \pi r l + \pi r^2 \Rightarrow \pi x \times 2x + \pi x^2 = 3\pi x^2$ <p>Volume of cone</p> $h = \frac{x}{\tan 30^\circ} = \sqrt{3}x$ $\text{Vol} = \frac{1}{3} \pi r^2 h \Rightarrow V = \frac{\sqrt{3}}{3} \pi x^3$ $\frac{dA}{dt} = 10 \text{ (cm}^2 / \text{s)} \quad \frac{dV}{dx} = \sqrt{3} \pi x^2, \frac{dA}{dx} = 6\pi x$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dA}{dt} \times \frac{dx}{dA}$ $\Rightarrow \frac{dV}{dt} = \sqrt{3} \pi x^2 \times 10 \times \frac{1}{6\pi x} = \frac{5\sqrt{3}}{3} x \Rightarrow 10\sqrt{3} \text{ (cm}^3 / \text{s)}$	$10\sqrt{3} \text{ (cm}^3 / \text{s)}$	<p>B1</p> <p>M1A1</p> <p>B1</p> <p>M1A1</p> <p>B1M1</p> <p>M1</p> <p>M1A1</p> <p>(11)</p>
		Total	100

9. Calculus

SAMs Question 11 Paper 2

A good place to start is to work out the Chain Rule to see what is needed.

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dS} \times \frac{dS}{dt}$$

This will score the M mark wherever it appears (correctly!)

Let the total surface area of the cone be S .

$$\frac{dS}{dt} = 10 \text{ cm/s}^2$$

B1

First part – deal with the surface area. Do not forget the base!

$$S = \pi x l + \pi x^2 \quad \text{but} \quad l = \frac{x}{\sin 30} = \frac{x}{0.5} = 2x \quad \text{B1}$$

$$\text{so} \quad S = 2\pi x^2 + \pi x^2 = 3\pi x^2 \quad \text{M1A1}$$

$$\text{and} \quad \frac{dS}{dx} = 6\pi x \quad \text{M1}$$



9. Calculus

SAMs Question 11 Paper 2

Model answer continued

Second part – deal with the volume

$$h = \frac{x}{\tan 30} = \sqrt{3}x \quad \text{B1}$$

$$V = \frac{1}{3}\pi x^2 h = \frac{\sqrt{3}}{3}\pi x^3 \Rightarrow \frac{dV}{dx} = \sqrt{3}\pi x^2 \quad \text{M1A1}$$

Final part – apply Chain Rule

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dS} \times \frac{dS}{dt} \quad \text{M1}$$

$$\Rightarrow \frac{dV}{dt} = \sqrt{3}\pi x^2 \times \frac{1}{6\pi x} \times 10 = \frac{5\sqrt{3}}{3}x = 10\sqrt{3} \text{ cm}^3/\text{s} \quad \text{M1A1}$$

Don't forget that an EXACT answer is required.



Pearson
Edexcel

10. Trigonometry

A	Radian measure including use for arc length and area of sector
B	The three basic trigonometrical ratios of angles of any magnitude (in radians or degrees) and their graphs
C	Applications to simple problems in two or three dimensions. (Including angles between a line and a plane and between two planes)
D	Use of sine and cosine formulae
E	The identity $\cos^2 A + \sin^2 A = 1$
F	The use of the basic addition formulae of trigonometry
H	The solution of simple trigonometric equations for a given interval



10. Trigonometry

There is no new content here. However, the cosine rule formula will be given in the formula sheet, but sine rule and the area of a triangle $A = \frac{1}{2}ab \sin C$ will be expected to be known.

$\tan A = \frac{\sin A}{\cos A}$ will be given on the formula sheet.

The summation formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$ will be given in the formula sheet.

There is no new content in **10H**, but we have clarified the specification to explicitly include equations such as;

$$\sin(3x - 30^\circ) = \frac{1}{4} \quad \text{for} \quad -90^\circ \leq x \leq 90^\circ$$



10. Trigonometry

SAMs Question 2 Paper 1

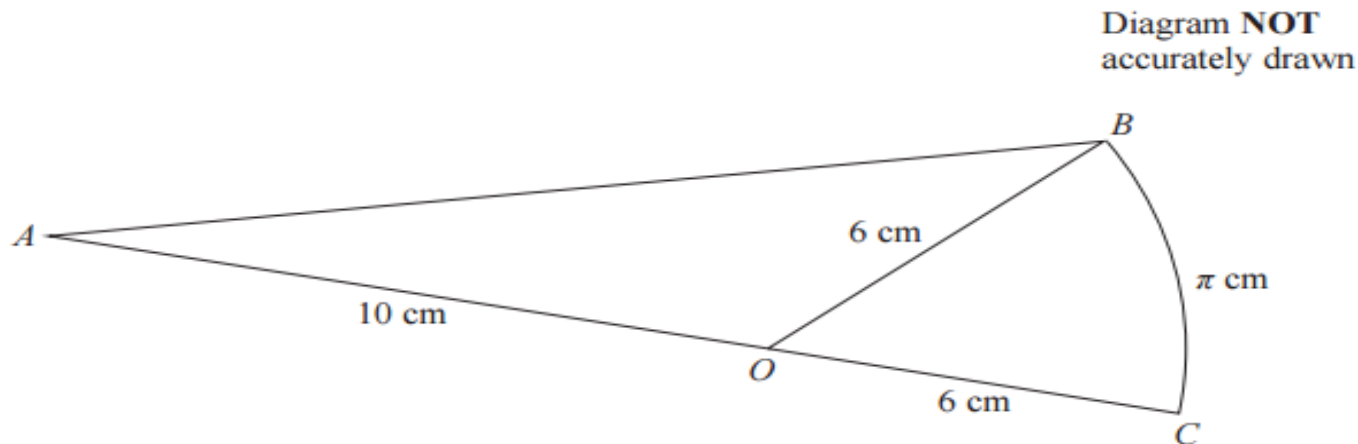


Figure 1

Figure 1 shows a shape ABC in which AOB is a triangle, AOC is a straight line and OBC is a sector of a circle with centre O .

$AO = 10$ cm, $OC = OB = 6$ cm and the length of arc $BC = \pi$ cm.

Find, to 3 significant figures,

(a) the length of AB ,

(3)

(b) the area of the shape ABC .

(3)

This question combines Spec **10A** and **10D**



10. Trigonometry

Question 5 Paper 1 June 2017

5 In triangle ABC , $AB = 10$ cm, $BC = 7$ cm and angle $BAC = 40^\circ$

(a) Find, in degrees to the nearest 0.1° , the two possible sizes of angle ACB .

(4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC .

(4)

Extracts from the Principal Examiner's report:

- More successful candidates sensibly started with a sketch, although surprisingly many attempted the question without.
- Where candidates lost marks on this part it was generally for not proceeding to find the value of obtuse angle C by subtracting their previous answer from 180° , or not rounding correctly to 1 decimal place as specified.
- Too many candidates did not understand difference in its mathematical sense as the result of subtraction and achieved only M1 M0 A1 A0 even where they found both possible lengths of AC correctly.
- Too many lost the final A0 by rounding prematurely, and $10.4 - 4.89 = 5.51$ cm was a very common final answer.

Notes		
(a)	M1	For using a correct sine rule either way around
	A1	For rearranging sine rule to give $C = \sin^{-1}\left(\frac{10 \sin 40}{7}\right)$ can be implied
	A1	$C = 66.7^\circ$
	A1	$C = (180 - 66.7) = 113.3^\circ$
(b)	M1	States or implies that triangle $CC'B$ is isosceles. This is all that is required for this mark.
	dM1	Use Cosine so length of half of base of triangle $BCC' = 7 \cos(66.67)$
	A1	$CC' = 2 \times 7 \cos(66.67)$ (follow through their acute angle ACB)
	A1	For 5.54 (cm)
	ALT	
	M1	For using an appropriate trig statement, ie., cosine rule or sine rule to find AC or AC'
	dM1	For a complete method to find the other length AC' or AC and the difference ($AC - AC' = \dots$)
	A1	Correct length of AC or AC' (either awrt 10.4 or 4.89)(cm) This A mark is dependent on the first M mark only.
	A1	For 5.54 (cm)
Total 8 marks		

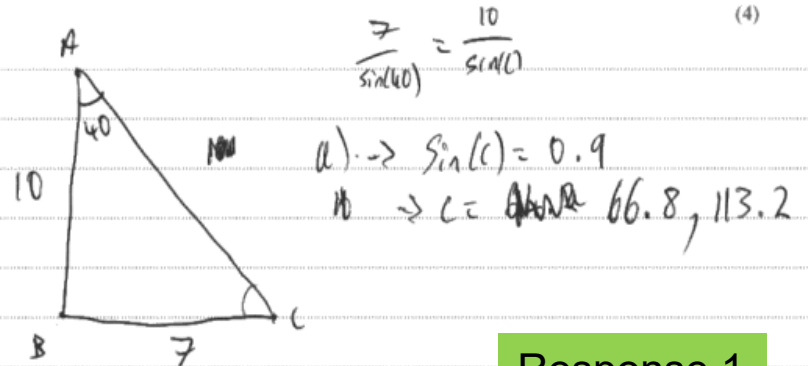
Marking Activity 3

Mark the following two responses noting how the candidates have lost marks needlessly

5 In triangle ABC , $AB = 10$ cm, $BC = 7$ cm and angle $BAC = 40^\circ$

(a) Find, in degrees to the nearest 0.1° , the two possible sizes of angle ACB . (4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC . (4)



Response 1

b). $L_1 - L_2$

$$AC^2 = 10^2 + 7^2 - 140 \cos(73.2)$$

$$\Rightarrow AC^2 = 108.109 \Rightarrow AC = 10.4$$

$$\Rightarrow AC^2 = 10^2 + 7^2 - 140 \cos(26.8)$$

$$\Rightarrow AC^2 = 24.0 \Rightarrow AC = 4.90$$

$$\Rightarrow 10.4 - 4.9 = 5.5$$

Response 2

$$\text{Sa. } \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\therefore \frac{\sin 40}{7} = \frac{\sin C}{10}$$

$$\frac{10 \sin 40}{7} = \sin C$$

$$\sin^{-1}\left(\frac{10 \sin 40}{7}\right) = C$$

$$66.7^\circ = \angle C$$

OR

$\triangle ABC$ could be isosceles

$$\therefore \frac{(180 - 40)}{2} = 70^\circ$$

$$\text{b. } a^2 = b^2 + c^2 - 2bc \cos A$$

$$\therefore a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$\therefore a = \sqrt{7^2 + 10^2 - 2(7)(10)(\cos 73.3^\circ)}$$

$$a = 9.6729...$$

~~$$\therefore a = \sqrt{7^2 + 10^2 - 2(7)(10)(\cos 70^\circ)}$$~~

$$\therefore a = 10 \text{ cm}$$

~~$$\therefore a = 10 \text{ cm}$$~~

~~$$\therefore a = 10 \text{ cm}$$~~

$$\therefore 10 - 9.6729... = \underline{\underline{0.327 \text{ cm}}}$$

Marking Activity 3 – mark allocation

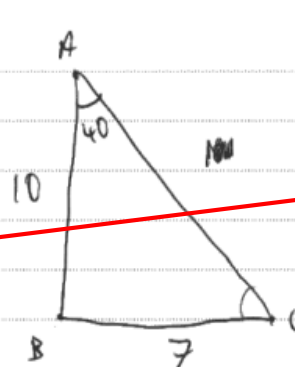
5 In triangle ABC , $AB = 10$ cm, $BC = 7$ cm and angle $BAC = 40^\circ$

(a) Find, in degrees to the nearest 0.1° , the two possible sizes of angle ACB .

(4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC .

(4)



$$\frac{7}{\sin(40)} = \frac{10}{\sin(C)}$$

$$a) \rightarrow \sin(C) = 0.9$$

$$\rightarrow C = 66.8, 113.2$$

Response 1

$$b) L_1 - L_2$$

$$AC^2 = 10^2 + 7^2 - 140 \cos(73.2)$$

$$\Rightarrow AC^2 = 108.109 \Rightarrow AC = 10.4$$

$$\Rightarrow AC^2 = 10^2 + 7^2 - 140 \cos(26.8)$$

$$\Rightarrow AC^2 = 24.0 \Rightarrow AC = 4.90$$

$$\Rightarrow 10.4 - 4.9 = 5.5$$

Correct method, but very premature rounding loses both A marks. $\sin^{-1} C = 0.91826...$ This candidate has used 0.9 M1A1A0A0

Again fully correct method. We allowed either 10.4 or 5.54 for one length of AC so this achieves M1A1M1A0

A fully correct method throughout – loses 3 A marks!

Response 2

$$\Delta a. \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\therefore \frac{\sin 40}{7} = \frac{\sin C}{10}$$

$$\frac{10 \sin 40}{7} = \sin C$$

$$\sin^{-1}\left(\frac{10 \sin 40}{7}\right) = C$$

$$66.7^\circ = \angle ACB$$

OR

ΔABC could be isosceles

$$\therefore \frac{(180 - 40)}{2} = 70^\circ$$

A fully correct method
for one of the angles,
and one correct angle
— M1A1A1A0

Incorrect
second angle

$$b. a^2 = b^2 + c^2 - 2bc \cos A$$

$$\therefore a = \sqrt{b^2 + c^2 - 2bc \cos A}$$

$$\therefore a = \sqrt{7^2 + 10^2 - 2(7)(10)(\cos 73.3^\circ)}$$

$$a = 9.6729...$$

$$\therefore a = \sqrt{7^2 + 10^2 - 2(7)(10)(\cos 70^\circ)}$$

$$\therefore a = 10 \text{ cm}$$

The formula for
cosine rule is
correct, but the
angle is not
M0A0A0A0

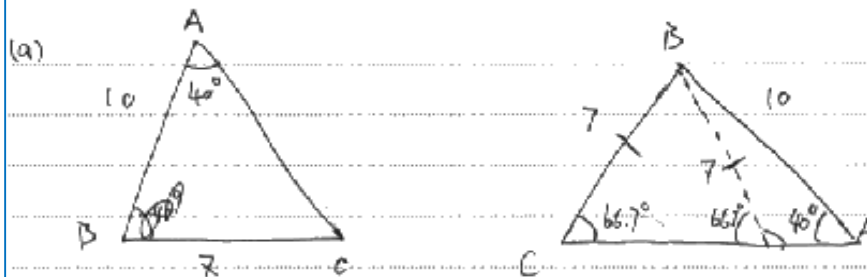
$$\therefore 10 - 9.6729... = 0.327 \text{ cm}$$

A model answer.
Note the excellent sketch.

5 In triangle ABC , $AB = 10$ cm, $BC = 7$ cm and angle $BAC = 40^\circ$

(a) Find, in degrees to the nearest 0.1° , the two possible sizes of angle ACB . (4)

(b) Find, in cm to 3 significant figures, the difference between the two possible lengths of AC . (4)



$$\frac{7}{\sin 40^\circ} = \frac{10}{\sin C}$$

$$\sin C = \frac{10 \sin 40^\circ}{7}$$

$$\text{angle } C = \arcsin\left(\frac{10 \sin 40^\circ}{7}\right) \approx 66.7^\circ$$

because it is the sin value
 \therefore another value is $180^\circ - 66.7^\circ = 113.3^\circ$
 $\angle ACB = 66.7^\circ$ or $\angle ACB = 113.3^\circ$

(b) $180 - 66.7^\circ \times 2 = 46.6^\circ$ let the difference be x

$$\frac{7}{\sin 66.7^\circ} = \frac{x}{\sin 46.6^\circ}$$

$$x = \frac{7 \sin 46.6^\circ}{\sin 66.7^\circ} \approx 5.54 \text{ cm}$$

An example of a beautiful and sophisticated solution to part (b)

10. Trigonometry

SAMs Question 3 Paper 1

3 Solve, in degrees to 1 decimal place, for $0 \leq \theta < 180$

$$2 \cos(2\theta + 30)^\circ + \tan(2\theta + 30)^\circ = 0 \quad (6)$$

This question combines specifications **10D**, **10E**, **10F** and **10H** and also quadratic equations **2A**.

Note that the identity $\tan A = \frac{\sin A}{\cos A}$ (which was also a hint to the

direction towards the solution) is no longer given in the question.

This identity is now found in the formula sheet at the front of the paper.



10. Trigonometry

SAMs Question 3 Paper 1

3 Solve, in degrees to 1 decimal place, for $0 \leq \theta < 180$

$$2 \cos(2\theta + 30)^\circ + \tan(2\theta + 30)^\circ = 0$$

(6)

Please remind your students that it is very important to find **all** the values for what will be $\sin(2\theta + 30^\circ)$ before subtracting 30° and dividing by 2.

That is;

$$\sin(2\theta + 30^\circ) = 1.2807..., 0.7807$$

$$2\theta + 30^\circ = 51.331..., 231.331..., 308.668...$$

$$\theta = 100.7^\circ, 139.3^\circ$$



10. Trigonometry

SAMs Question 10 Paper 1

Diagram for the question

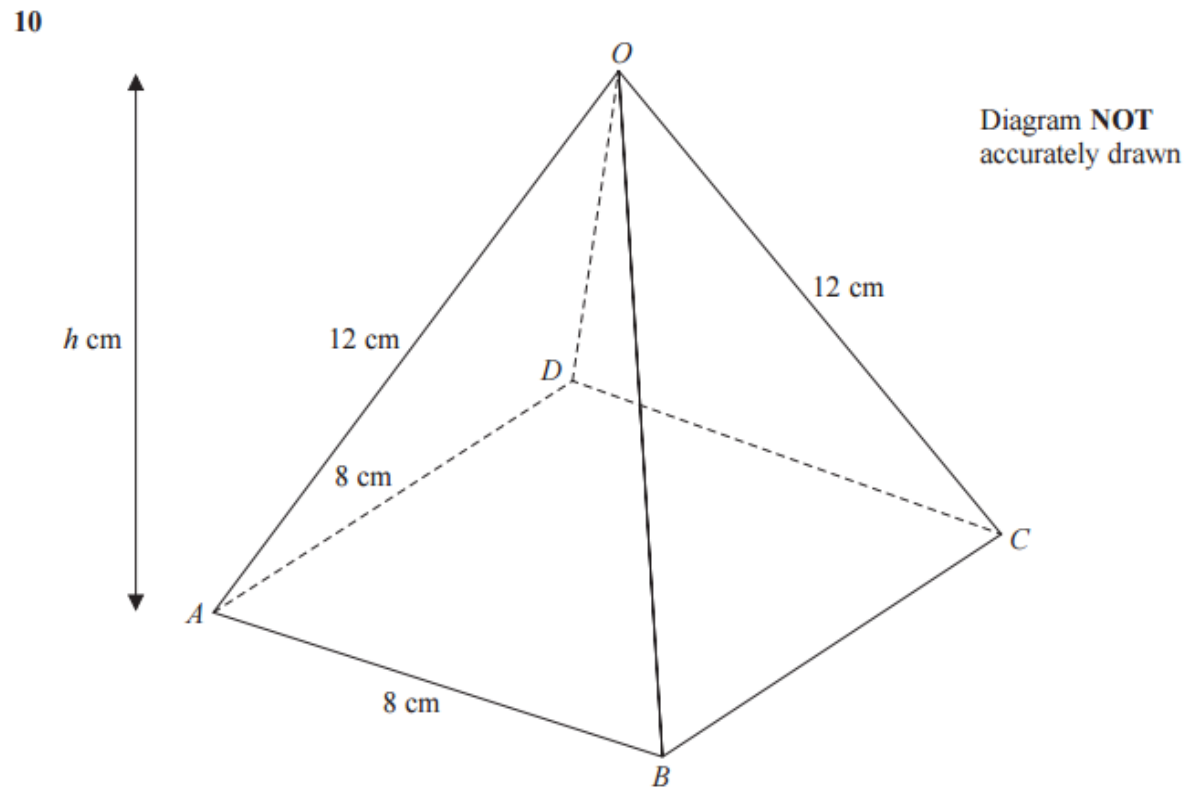


Figure 2



10. Trigonometry

SAMs Question 10 Paper 1 Text of the question

This question is ramped in difficulty and gradually moves from Grade 4 to Grade 9

Figure 2 shows a right pyramid $ABCD O$ with a horizontal square base of side 8 cm. The vertical height of the pyramid is h cm and $OA = OB = OC = OD = 12$ cm.

(a) Find the exact value of h .

(3)

(b) Find, to 1 decimal place, the size of the angle between OA and the plane $ABCD$.

(2)

(c) Find, to 1 decimal place, the size of the angle between the plane AOB and the plane $ABCD$.

(2)

The midpoint of OA is P and Q is the point on BC such that $BQ : QC = 3 : 1$

(d) Show that $PQ = 4\sqrt{5}$ cm.

Parts (d) and (e) are examples
of parts of a question aiming
for Grade 9 (4)

(e) Find, to 1 decimal place, the size of angle PQA .

(4)



10. Trigonometry

SAMs Question 5 Paper 2

5 (a) Show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$ (2)

(b) Hence express $2 \sin 5x \sin 3x$ in the form $\cos mx - \cos nx$ where m and n are integers, giving the value of m and the value of n , (1)

(c) (i) Find $\int 4 \sin 5\theta \sin 3\theta \, d\theta$

(ii) Hence evaluate $\int_0^{\frac{\pi}{6}} 4 \sin 5\theta \sin 3\theta \, d\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where a , b and c are integers.

(4)

This question tests specification **10G**, **9A** and **1C**.

Part (c) is an example of a question aiming for Grade 9



Grade 9 Questions



What does grade 9 look like?

Examples of questions and part questions that are targeting grade 9.

Questions in the SAMs papers are as follows;

Question 10 paper 1 parts (d) and (e)	3D Trigonometry	8 marks
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Question 11 paper 1 part (a)	Problem solving/integration	8 marks
------------------------------	-----------------------------	---------

Question 5 paper 2 part (c)	Integration of a trig function	4 marks
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Question 11 paper 2 whole question	Connected rates of change	11 marks
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Total 29 marks, so approximately 15% of the overall specification



Common Issues



Common Issues

1. Incorrect answers without working will lose all marks associated with the question.

Encourage your students to show all working.

2. Please emphasize the importance of reading the demands in the question. If a question requires a simplified answer then that is what the candidate should leave as their final answer.

3. Candidates must read rounding requirements carefully and give their answers to the required degree of accuracy, and round **only** at the end of the question.



Subject Features

Reviewed and updated in light of UK GCSE changes

Broaden numerical skills in both a mathematical way and real-life situations

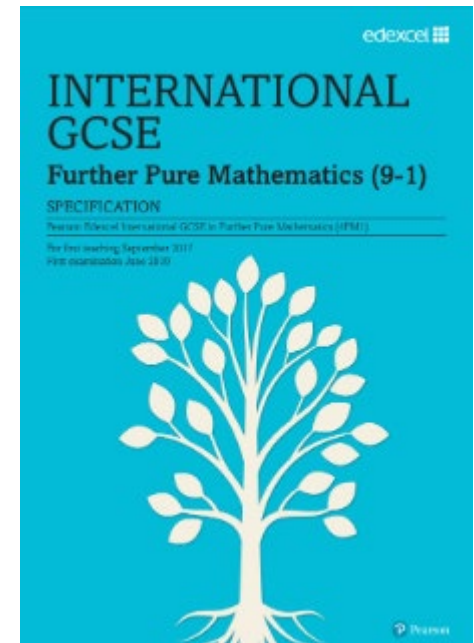
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- Questions tagged against unit, topic and assessment objective or simply choose a whole past paper.
- Use existing mark schemes for accurate marking.
- Use examiner report for insight.
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- Use the results to understand where students need more support, informing teaching strategies.



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Subject Advisor details

Your subject advisor is **Graham Cumming**

Phone: **+44 (0)20 7010 2174**

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Email: TeachingMaths@pearson.com

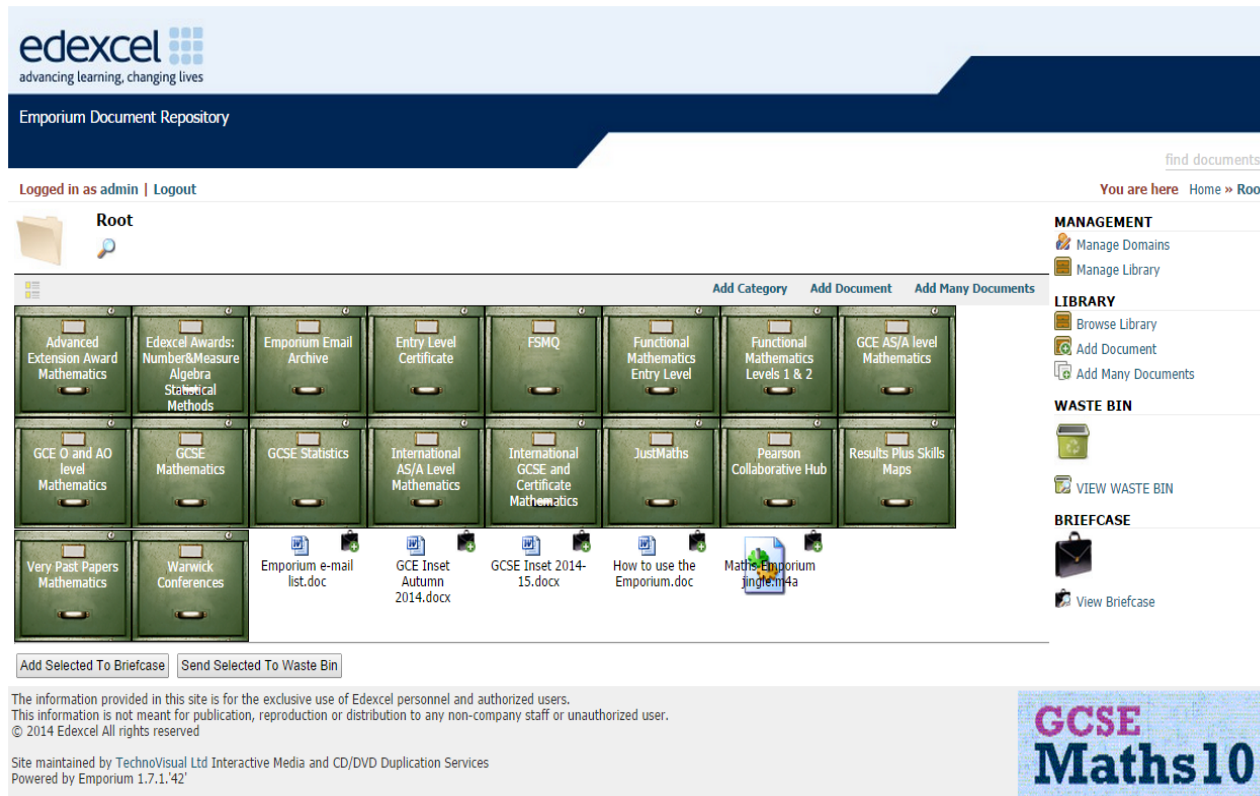


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